

An Adjusted Human Development Index: Robust Country Rankings with Respect to the Choice of Fixed Maximum and Minimum Indicator Values

RAMAKRUSHNA PANIGRAHI and SASHI SIVRAMKRISHNA

Ramakrushna Panigrahi is a Faculty Member at Icfai Business School, Bangalore

Sashi Sivramkrishna is Director, Foundation to Aid Industrial Recovery

Introduction

Three important objectives of the Human Development Index (HDI) can be found in the United Nations Development Programme (1999) *Human Development Report*.

1. It breaks the dominance of the Gross National Product as *the* index of development. A single index — just one number — has proved useful in this achievement, which may not have been possible with any set of tables (Sen, 1999).
2. It shows the distance that a country has traveled towards the maximum possible value of 1.
3. It allows comparisons with other countries, both in terms of country rank as well as in terms of HDI values.

The United Nations Development Programme (UNDP) HDI is based on three indicators; longevity, educational attainment, and standard of living, each assigned equal weights. Longevity is indexed by Life Expectancy at Birth (LEB), education attainment by Adult Literacy Rate, and Gross Enrolment Ratio combined in a 2:1 ratio to yield the overall education level (EDN) and standard of living indexed by Purchasing Power Parity Gross Domestic Product per-capita (GDP). For each country j , the index values for health (L_j), education (E_j) and income (G_j), are computed as follows:

$$L_j = \frac{LEB_j - LEB_{\min}}{LEB_{\max} - LEB_{\min}}$$

where $LEB_{\max}$ and $LEB_{\min}$ are set at 85 years and 25 years, respectively.

$$E_j = \frac{EDN_j - EDN_{\min}}{EDN_{\max} - EDN_{\min}}$$

where EDN_{max} and EDN_{min} are set at 100% and 0%, respectively.

$$G_j = \frac{\log(GDP_j) - \log(GDP_{min})}{\log(GDP_{max}) - \log(GDP_{min})}$$

where GDP_{max} and GDP_{min} are set at \$40,000 and \$100, respectively.

The HDI value for each country j is then given by:

$$HDI_j = (L_j + E_j + G_j)/3$$

Country rankings are based on the values of HDI, with higher values giving countries a better rank.

The fixed maximum and minimum values (limits) for each indicator

The estimation of index values for each indicator using the fixed maximum and minimum values, also referred to as 'limits', are useful in meeting the second objective stated earlier (i.e. they tell us how far the country is from the 'minimum' and how far it has to go before it reaches the 'maximum'), or as Ray (1998) puts it, "ultimate bliss".

It is evident from the equations already specified to calculate L_j , E_j and G_j that the choice of the maximum and minimum fixed limits will affect HDI values for each country. Kelley (1991) points this out:

... the HDI turns out to be quite sensitive to the choice of maximum life expectancy. As an experiment, I set the adequate/desirable life expectancy at age 73¹ and evaluated the impact. This raised 22 countries from 'low' to medium human development and raised another ten countries from 'medium' to 'high' human development.

We make two comments on this observation. First, reducing LEB_{max} from 78 to 73 seems quite unjustifiable when some countries have already achieved a LEB of 78 years.² Second, given a fixed cut-off HDI value to categorize countries as 'low', 'medium' and 'high' development, it is quite obvious that reducing LEB_{max} will raise HDI values for all countries, and consequently some countries will move from a lower to a higher level of development.

Referring to Dasgupta (1993), Atkinson *et al.* (1999) make a similar argument of the sensitivity of HDI *values* to a change in limits:

... if the minimum value of X_i ³ was to change, the HDI of all countries would change, even though nothing need have happened in those countries to alter the 'output' of health, education or the 'input' of income.

However, what is interesting to note is not the mere sensitivity of HDI *values* to the choice of limits, but the sensitivity of *HDI-based ranking* to the choice of limits. The sensitivity to limits we refer to is not what Kelley (1991) points out, where countries move from one level of development to another with their new HDI values, but the possibility of an *interchanging*

or re-ordering of HDI-based country ranks. In other words, for given indicator values, with one set of limits we may have a situation where country 1 is ranked better than country 2; but with a different set of limits, country 2 is ranked better than country 1. If so, an HDI-based country ranking or ordering is not robust or consistent with respect to the choice of limits.

Understanding the effect of limits on HDI-based country ranking

We illustrate our argument with a hypothetical two-country example. Table 1 presents the HDI and HDI-based ranks, for each country (country A and country B) with the following limits: $LEB_{max} = 85$ years and $LEB_{min} = 25$ years, $EDN_{max} = 100\%$ and $EDN_{min} = 0\%$, and $GDP_{max} = \$40,000$ and $GDP_{min} = \$100$.

Suppose we now change the limits to $LEB_{max} = 82$ years and $LEB_{min} = 42$ years, with the limits for EDN and GDP remaining unchanged. HDI values will obviously change. However, and more interestingly, we see from Table 2 that the HDI-based ranking of countries A and B also change.

Let us try to understand why HDI-based rankings change in this example.

1. To give equal importance to health, education and standard of living in human development, the HDI is constructed as a simple average of the health index, the education index and the income index. The *explicit* equal weights assigned to each indicator conceals the *implicit* assumption that 0.6 years of LEB, 1% in education level and a real adjusted income⁴ of \$150 for country A (denoted as \$150_A) and \$123 for country B (denoted as \$123_B) are 'equal'. This is so because an increase in any one of these indicators by these respective amounts would make an equal contribution of 0.01 to the individual indicator index and 0.0033 to HDI value. In other words, the *equality* between health, education and income is actually fixed by the maximum and minimum values chosen for each indicator. A change in the limits for LEB from 85/25 to 82/42 then means a change in the stated terms of equality from '0.6 years LEB = 1%

TABLE 1. HDI and HDI-based ranking based on UNDP limits

	LEB	EDN	GDP	L_j	E_j	G_j	HDI _j	Rank _j
Country A	65.4	90	2500	0.67	0.90	0.5373	0.7024	1
Country B	72.8	79	2000	0.80	0.79	0.50	0.6967	2

TABLE 2. HDI and HDI-based ranking with changed LEB limits

	LEB	EDN	GDP	L_j	E_j	G_j	HDI _j	Rank _j
Country A	65.4	90	2500	0.585	0.90	0.5373	0.6741	2
Country B	72.8	79	2000	0.770	0.79	0.50	0.6867	1

EDN = \$150_A or \$123_B to '0.4 years LEB = 1% EDN = \$150_A or \$123_B'. The increase in the *implicit* weight to LEB with the change in its limits enables country B to improve its rank from 2 to 1.

2. A careful look at Tables 1 and 2 also reveals that a change in LEB limits not only changes in the absolute LEB index value for each country, but also the *spread* between the LEB index values, from 0.13 (= 0.80 – 0.67) to 0.185 (= 0.770 – 0.585). This causes the switch in HDI-based ranks of country A and country B with the change in LEB limits.

We use these important insights to develop an adjusted HDI (in later section 'The adjusted HDI'), which gives country ranks that are invariant to the choice of limits.

Borda Count-based country ranking

Before presenting the adjusted HDI, we draw attention to a ranking method that is independent of the choice of fixed limits, based on the Borda Count⁵ (Dasgupta, 1993; Atkinson *et al.*, 1999). This independence of ranking from the choice of limits gives equal weights to health, education and income without any implicit equality in values of LEB, EDN and GDP.⁶ Column (C7) in Table 3 gives country ranks based on Borda Count. The difference in country ranks based on the UNDP-HDI method and the Borda Count method can be ascertained from Columns C6 and C7 in Table 3.

A Borda Count-based ranking tells us little about the difference in country achievements, both in absolute terms as well as relative to other countries. Consider a ranking of Canada, Norway, the United States and Sierra Leone. The Borda Rank for Canada may be 1 out of 4, that for Norway 3 out of 4 and that of Sierra Leone 4 out of the four countries ranked. Does that mean Norway is closer in development to Sierra Leone than to Canada?

Also, from Table 3, we find that the United States moves from a HDI-based ranking of 2 to 11.⁷ A slightly lower value in LEB and EDN values for the United States as compared with Canada, Japan and European countries forces down its individual indicator rankings to 24 and 10, respectively. On the contrary, the score that the United States receives from its income rank does not reflect the relatively large difference in income between the United States and some of these countries.

In addition to these limitations as a ranking method, the Borda Count does not meet the objectives of the HDI stated in the 'Introduction'.

The adjusted HDI

Based on the earlier third section 'Understanding the effect of limits on HDI-based country ranking', we propose an adjusted HDI (AHDI) that, like the HDI, meets the objectives listed in the 'Introduction', and at the same time gives country rankings that are invariant to changes in the fixed limits.

An Adjusted HDI

TABLE 3. HDI, Borda Count rank and AHDH for all countries based on UNDP (1999) data

C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
Albania	0.797	0.793	0.510	0.700	100	97	0.698	99	0.624	102	2	0.623	99	0
Algeria	0.732	0.629	0.634	0.665	109	105	0.669	108	0.588	111	2	0.597	108	0
Angola	0.358	0.390	0.444	0.397	161	158	0.399	159	0.354	160	-1	0.356	159	0
Antigua and Barbuda	0.833	0.887	0.763	0.828	38	38	0.828	38	0.738	38	0	0.739	38	0
Argentina	0.798	0.907	0.774	0.826	39	37	0.825	39	0.739	37	-2	0.736	39	0
Armenia	0.758	0.899	0.528	0.728	86	75	0.722	89	0.654	83	-3	0.644	89	0
Australia	0.887	0.993	0.886	0.922	7	4	0.921	7	0.824	6	-1	0.822	7	0
Austria	0.867	0.947	0.901	0.905	16	17	0.905	16	0.807	16	0	0.808	16	0
Azerbaijan	0.748	0.879	0.457	0.695	101	100	0.688	102	0.625	101	0	0.614	102	0
Bahamas	0.813	0.885	0.854	0.851	31	36	0.852	31	0.759	31	0	0.761	31	0
Bahrain	0.798	0.845	0.852	0.832	37	42	0.834	36	0.741	36	-1	0.744	36	0
Bangladesh	0.552	0.383	0.392	0.442	150	150	0.447	150	0.388	151	1	0.399	150	0
Barbados	0.857	0.917	0.799	0.858	29	25	0.858	29	0.765	29	0	0.765	29	0
Belarus	0.717	0.927	0.648	0.764	60	59	0.758	61	0.687	56	-4	0.676	61	0
Belgium	0.870	0.993	0.906	0.923	5	3	0.922	5	0.825	4	-1	0.823	5	0
Belize	0.828	0.740	0.628	0.732	83	75	0.735	80	0.649	88	5	0.656	80	0
Benin	0.473	0.366	0.424	0.421	155	153	0.426	153	0.371	156	1	0.380	153	0
Bhutan	0.595	0.335	0.448	0.459	145	143	0.468	143	0.400	147	2	0.418	143	0
Bolivia	0.607	0.791	0.561	0.653	112	115	0.648	112	0.587	112	0	0.578	112	0
Botswana	0.373	0.729	0.725	0.609	122	113	0.604	124	0.552	119	-3	0.539	124	0
Brazil	0.697	0.827	0.696	0.740	79	93	0.738	77	0.663	78	-1	0.658	77	0
Brunei Darussalam	0.842	0.841	0.951	0.878	25	31	0.883	25	0.780	25	0	0.788	25	0
Bulgaria	0.768	0.888	0.616	0.757	63	60	0.753	66	0.679	63	0	0.672	66	0
Burkina Faso	0.323	0.205	0.386	0.305	171	171	0.311	171	0.266	171	0	0.278	171	0
Burundi	0.290	0.374	0.307	0.324	170	172	0.322	170	0.291	170	0	0.288	170	0
Cambodia	0.473	0.643	0.427	0.514	137	137	0.509	138	0.464	137	0	0.455	138	0
Cameroon	0.495	0.621	0.491	0.536	134	132	0.533	134	0.481	135	1	0.476	134	0
Canada	0.900	0.990	0.904	0.931	1	1	0.931	1	0.831	1	0	0.831	1	0
Cape Verde	0.732	0.730	0.567	0.676	106	108	0.676	105	0.602	106	0	0.603	105	0
Central African Republic	0.332	0.369	0.432	0.378	165	163	0.379	165	0.336	165	0	0.338	165	0
Chad	0.370	0.432	0.379	0.394	162	162	0.393	162	0.352	162	0	0.351	162	0
Chile	0.832	0.891	0.809	0.844	34	34	0.844	33	0.753	34	0	0.754	33	0
China	0.747	0.783	0.575	0.701	98	102	0.700	97	0.626	100	2	0.625	97	0
Colombia	0.757	0.843	0.704	0.768	57	62	0.767	58	0.686	57	0	0.684	58	0
Comoros	0.563	0.499	0.455	0.506	139	138	0.508	139	0.448	138	-1	0.454	139	0
Congo	0.393	0.739	0.465	0.532	135	135	0.523	135	0.486	132	-3	0.467	135	0
Congo, Democratic Republic of	0.430	0.643	0.363	0.479	141	143	0.472	142	0.434	141	0	0.421	142	0
Costa Rica	0.850	0.854	0.701	0.802	45	43	0.802	44	0.714	45	0	0.716	44	0
Cote D'Ivoire	0.362	0.417	0.486	0.422	154	147	0.423	154	0.376	154	0	0.378	154	0
Croatia	0.793	0.875	0.649	0.772	55	54	0.770	55	0.691	55	0	0.687	55	0
Cuba	0.845	0.879	0.573	0.766	58	55	0.763	59	0.684	58	0	0.681	59	0
Cyprus	0.880	0.903	0.827	0.870	26	24	0.871	26	0.775	26	0	0.778	26	0
Czech Republic	0.815	0.907	0.777	0.833	35	33	0.832	37	0.744	35	0	0.743	37	0
Denmark	0.845	0.957	0.913	0.905	15	16	0.905	17	0.808	15	0	0.808	17	0
Djibouti	0.423	0.392	0.424	0.413	157	157	0.416	156	0.366	159	2	0.371	156	0
Dominica	0.817	0.883	0.629	0.776	53	48	0.773	53	0.694	54	1	0.690	53	0
Dominican Republic	0.760	0.771	0.647	0.726	88	88	0.726	87	0.647	90	2	0.648	87	0
Ecuador	0.742	0.848	0.651	0.747	72	78	0.744	71	0.669	73	1	0.664	71	0
Egypt	0.688	0.591	0.570	0.617	120	122	0.620	119	0.546	122	2	0.554	119	0
El Salvador	0.735	0.727	0.561	0.674	107	111	0.674	107	0.600	108	1	0.602	107	0

Table 3. Continued.

C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
Equatorial														
Guinea	0.417	0.746	0.484	0.549	131	130	0.540	133	0.500	131	0	0.482	133	0
Eritrea	0.430	0.257	0.351	0.346	167	167	0.352	167	0.302	169	2	0.314	167	0
Estonia	0.728	0.930	0.661	0.773	54	53	0.767	56	0.695	53	-1	0.685	56	0
Ethiopia	0.305	0.316	0.272	0.298	173	173	0.298	173	0.265	172	-1	0.266	173	0
Fiji	0.795	0.879	0.615	0.763	61	58	0.760	60	0.683	61	0	0.678	60	0
Finland	0.863	0.990	0.886	0.913	13	15	0.912	13	0.816	12	-1	0.814	13	0
France	0.885	0.967	0.900	0.917	11	9	0.918	10	0.819	10	-1	0.819	10	0
Gabon	0.457	0.641	0.722	0.607	124	114	0.606	123	0.544	123	-1	0.541	123	0
Gambia	0.367	0.357	0.449	0.391	163	159	0.394	161	0.347	164	1	0.352	161	0
Georgia	0.795	0.897	0.497	0.729	84	69	0.723	88	0.654	81	-3	0.646	88	0
Germany	0.870	0.953	0.895	0.906	14	13	0.906	14	0.808	14	0	0.809	14	0
Ghana	0.583	0.583	0.467	0.544	133	133	0.544	132	0.485	133	0	0.486	132	0
Greece	0.885	0.907	0.809	0.867	27	22	0.868	27	0.772	27	0	0.775	27	0
Grenada	0.783	0.900	0.648	0.777	52	49	0.773	52	0.696	52	0	0.690	52	0
Guatemala	0.650	0.601	0.620	0.623	117	119	0.627	118	0.553	118	1	0.560	118	0
Guinea	0.358	0.346	0.490	0.398	160	155	0.402	158	0.352	161	1	0.359	158	0
Guinea-Bissau	0.333	0.337	0.359	0.343	168	169	0.345	168	0.305	167	-1	0.308	168	0
Guyana	0.657	0.867	0.579	0.701	99	102	0.695	101	0.632	98	-1	0.620	101	0
Haiti	0.478	0.385	0.424	0.429	152	150	0.433	151	0.379	153	1	0.387	151	0
Hong Kong,														
China (SAR)	0.892	0.833	0.917	0.880	24	26	0.886	23	0.780	24	0	0.791	23	0
Honduras	0.740	0.665	0.517	0.641	114	117	0.642	113	0.568	115	1	0.573	113	0
Hungary	0.765	0.907	0.714	0.795	47	44	0.792	48	0.713	46	-1	0.707	48	0
Iceland	0.900	0.950	0.904	0.918	9	7	0.919	9	0.818	11	2	0.821	9	0
India	0.627	0.540	0.470	0.546	132	134	0.548	130	0.483	134	2	0.489	130	0
Indonesia	0.668	0.780	0.593	0.680	105	107	0.677	104	0.610	105	0	0.605	104	0
Iran, Islamic														
Republic of	0.737	0.729	0.678	0.715	95	98	0.716	91	0.635	96	1	0.639	91	0
Iraq	0.623	0.557	0.578	0.586	125	128	0.590	126	0.519	127	2	0.526	126	0
Ireland	0.855	0.953	0.890	0.899	20	20	0.900	20	0.803	18	-2	0.803	20	0
Israel	0.880	0.903	0.868	0.884	23	23	0.886	24	0.787	23	0	0.791	24	0
Italy	0.887	0.929	0.887	0.901	19	14	0.902	18	0.802	20	1	0.805	18	0
Jamaica	0.830	0.780	0.591	0.734	82	77	0.734	81	0.652	85	3	0.655	81	0
Japan	0.917	0.943	0.915	0.925	4	4	0.927	3	0.823	7	3	0.828	3	0
Jordan	0.752	0.801	0.591	0.715	94	96	0.713	93	0.638	95	1	0.636	93	0
Kazakhstan	0.710	0.913	0.596	0.740	78	72	0.733	83	0.666	75	-3	0.654	83	0
Kenya	0.450	0.695	0.413	0.520	136	139	0.512	137	0.471	136	0	0.457	137	0
Korea,														
Republic of	0.790	0.948	0.820	0.853	30	30	0.850	32	0.764	30	0	0.759	32	0
Kuwait	0.848	0.726	0.924	0.833	36	41	0.841	35	0.735	40	4	0.751	35	0
Kyrgyzstan	0.710	0.877	0.520	0.702	97	101	0.695	100	0.632	97	0	0.621	100	0
Lao People's														
Democratic														
Republic	0.470	0.574	0.428	0.491	140	142	0.488	140	0.440	140	0	0.436	140	0
Latvia	0.723	0.897	0.613	0.744	74	78	0.739	76	0.669	71	-3	0.660	76	0
Lebanon	0.748	0.816	0.682	0.749	69	81	0.748	68	0.669	72	3	0.667	68	0
Libyan Arab														
Jamahiriya	0.750	0.817	0.702	0.756	65	72	0.756	64	0.675	65	0	0.674	64	0
Lithuania	0.748	0.910	0.625	0.761	62	57	0.756	63	0.683	60	-2	0.675	63	0
Lesotho	0.517	0.742	0.488	0.582	127	127	0.576	127	0.526	125	-2	0.514	127	0
Luxembourg	0.862	0.890	0.957	0.903	17	21	0.906	15	0.803	19	2	0.809	15	0
Macedonia, Tlyr	0.802	0.860	0.579	0.747	71	65	0.744	72	0.668	74	3	0.664	72	0
Madagascar	0.542	0.443	0.372	0.452	147	149	0.455	146	0.400	146	-1	0.406	146	0
Malawi	0.238	0.635	0.327	0.400	159	163	0.388	163	0.370	158	-1	0.347	163	0
Malaysia	0.783	0.788	0.734	0.769	56	60	0.770	54	0.684	59	3	0.688	54	0

An Adjusted HDI

TABLE 3. Continued.

C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
Maldives	0.658	0.885	0.602	0.715	93	95	0.708	95	0.645	91	-2	0.632	95	0
Moldova, Republic of	0.708	0.889	0.452	0.683	104	104	0.675	106	0.616	104	0	0.602	106	0
Mali	0.472	0.320	0.334	0.375	166	165	0.380	164	0.329	166	0	0.339	164	0
Malta	0.870	0.867	0.815	0.851	32	32	0.853	30	0.757	32	0	0.761	30	0
Mauritania	0.475	0.393	0.476	0.448	149	141	0.452	147	0.395	150	1	0.404	147	0
Mauritius	0.773	0.763	0.757	0.764	59	63	0.767	57	0.679	62	3	0.685	57	0
Mexico	0.787	0.834	0.739	0.787	50	52	0.787	50	0.701	50	0	0.702	50	0
Mongolia	0.680	0.743	0.429	0.618	119	126	0.613	121	0.553	117	-2	0.548	121	0
Morocco	0.693	0.469	0.584	0.582	126	123	0.591	125	0.510	129	3	0.527	125	0
Mozambique	0.337	0.353	0.334	0.341	169	169	0.342	169	0.304	168	-1	0.305	169	0
Myanmar	0.585	0.741	0.415	0.580	128	130	0.574	128	0.523	126	-2	0.512	128	0
Namibia	0.457	0.805	0.653	0.638	115	106	0.631	116	0.579	114	-1	0.563	116	0
Nepal	0.538	0.451	0.399	0.463	144	146	0.465	145	0.409	145	1	0.415	145	0
Netherlands	0.882	0.987	0.893	0.921	8	11	0.920	8	0.822	8	0	0.821	8	0
New Zealand	0.865	0.977	0.861	0.901	18	19	0.900	19	0.805	17	-1	0.803	19	0
Nicaragua	0.715	0.633	0.500	0.616	121	121	0.617	120	0.546	121	0	0.551	120	0
Niger	0.392	0.145	0.357	0.298	172	168	0.307	172	0.256	173	1	0.274	172	0
Nigeria	0.418	0.577	0.370	0.455	146	153	0.450	149	0.411	144	-2	0.402	149	0
Norway	0.885	0.977	0.918	0.927	3	2	0.927	4	0.827	3	0	0.827	4	0
Oman	0.765	0.641	0.768	0.725	89	84	0.732	84	0.640	94	5	0.653	84	0
Pakistan	0.650	0.416	0.459	0.508	138	136	0.515	136	0.445	139	1	0.460	136	0
Panama	0.810	0.851	0.713	0.791	49	46	0.791	49	0.706	49	0	0.706	49	0
Papua New Guinea	0.548	0.615	0.547	0.570	129	129	0.570	129	0.509	130	1	0.508	129	0
Paraguay	0.743	0.829	0.615	0.729	85	91	0.726	86	0.652	84	-1	0.649	86	0
Peru	0.722	0.851	0.642	0.738	80	89	0.735	79	0.662	79	-1	0.656	79	0
Philippines	0.722	0.904	0.594	0.740	76	78	0.734	82	0.666	76	0	0.655	82	0
Poland	0.792	0.917	0.697	0.802	44	40	0.799	45	0.718	43	-1	0.713	45	0
Portugal	0.838	0.909	0.828	0.858	28	28	0.859	28	0.766	28	0	0.766	28	0
Qatar	0.778	0.770	0.892	0.814	41	50	0.819	42	0.722	42	1	0.731	42	0
Romania	0.748	0.879	0.628	0.752	68	67	0.748	69	0.674	66	-2	0.667	69	0
Russian Federation	0.693	0.917	0.630	0.747	73	66	0.740	75	0.673	68	-5	0.661	75	0
Rwanda	0.258	0.563	0.315	0.379	164	166	0.370	166	0.348	163	-1	0.330	166	0
Saint Kitts And Nevis	0.750	0.860	0.732	0.781	51	56	0.779	51	0.698	51	0	0.695	51	0
Saint Lucia	0.750	0.793	0.667	0.737	81	82	0.736	78	0.657	80	-1	0.657	78	0
Saint Vincent & Grenadines	0.800	0.807	0.626	0.744	75	69	0.744	73	0.663	77	2	0.664	73	0
Samoa (Western)	0.772	0.873	0.596	0.747	70	69	0.743	74	0.669	70	0	0.663	74	0
Sao Tome and Principe	0.650	0.690	0.487	0.609	123	125	0.607	122	0.544	124	1	0.542	122	0
Saudi Arabia	0.773	0.676	0.771	0.740	77	74	0.746	70	0.654	82	5	0.666	70	0
Senegal	0.455	0.347	0.476	0.426	153	147	0.432	152	0.375	155	2	0.385	152	0
Seychelles	0.767	0.763	0.735	0.755	66	67	0.757	62	0.671	69	3	0.676	62	0
Sierra Leone	0.203	0.322	0.235	0.254	174	174	0.251	174	0.230	174	0	0.224	174	0
Singapore	0.868	0.853	0.943	0.888	22	27	0.893	22	0.788	22	0	0.797	22	0
Slovakia	0.800	0.910	0.729	0.813	42	39	0.811	43	0.728	41	-1	0.724	43	0
Slovenia	0.823	0.913	0.796	0.844	33	29	0.844	34	0.754	33	0	0.753	34	0
Solomon Islands	0.778	0.567	0.524	0.623	118	112	0.629	117	0.548	120	2	0.561	117	0
South Africa	0.495	0.870	0.718	0.694	103	91	0.687	103	0.630	99	-4	0.613	103	0
Spain	0.885	0.955	0.846	0.895	21	18	0.895	21	0.798	21	0	0.799	21	0
Sri Lanka	0.802	0.825	0.537	0.721	90	86	0.718	90	0.644	93	3	0.641	90	0
Sudan	0.500	0.469	0.459	0.476	142	140	0.478	141	0.422	143	1	0.427	141	0

TABLE 3. Continued.

C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15
Suriname	0.752	0.860	0.658	0.757	64	64	0.754	65	0.677	64	0	0.673	65	0
Swaziland	0.587	0.760	0.586	0.644	113	117	0.640	114	0.579	113	0	0.571	114	0
Sweden	0.892	0.993	0.883	0.923	6	4	0.922	6	0.824	5	-1	0.823	6	0
Switzerland	0.893	0.923	0.923	0.913	12	8	0.916	12	0.813	13	1	0.818	12	0
Syrian Arab Republic	0.732	0.677	0.581	0.663	111	109	0.665	109	0.589	110	-1	0.594	109	0
Tajikistan	0.703	0.889	0.404	0.666	108	109	0.656	111	0.601	107	-1	0.586	111	0
Tanzania, United Republic of	0.382	0.587	0.293	0.421	156	159	0.413	157	0.382	152	-4	0.369	157	0
Thailand	0.730	0.828	0.702	0.753	67	83	0.752	67	0.673	67	0	0.671	67	0
Togo	0.397	0.558	0.451	0.469	143	145	0.465	144	0.422	142	-1	0.415	144	0
Trinidad and Tobago	0.813	0.872	0.705	0.797	46	45	0.796	46	0.711	47	1	0.710	46	0
Tunisia	0.742	0.680	0.663	0.695	102	99	0.698	98	0.616	103	1	0.623	98	0
Turkey	0.733	0.758	0.693	0.728	87	94	0.729	85	0.649	89	2	0.651	85	0
Turkmenistan	0.673	0.953	0.509	0.712	96	90	0.701	96	0.644	92	-4	0.626	96	0
Uganda	0.243	0.560	0.409	0.404	158	161	0.397	160	0.370	157	-1	0.354	160	0
Ukraine	0.730	0.917	0.515	0.721	91	84	0.713	92	0.649	87	-4	0.636	92	0
United Arab Emirates	0.830	0.729	0.877	0.812	43	46	0.819	41	0.718	44	1	0.731	41	0
United Kingdom	0.870	0.993	0.890	0.918	10	10	0.917	11	0.821	9	-1	0.818	11	0
United States	0.862	0.973	0.946	0.927	2	11	0.928	2	0.828	2	0	0.828	2	0
Uruguay	0.815	0.907	0.755	0.825	40	35	0.824	40	0.738	39	-1	0.736	40	0
Uzbekistan	0.708	0.913	0.539	0.720	92	87	0.713	94	0.649	86	-6	0.636	94	0
Vanuatu	0.707	0.583	0.592	0.627	116	120	0.632	115	0.554	116	0	0.564	115	0
Venezuela	0.790	0.837	0.748	0.792	48	51	0.792	47	0.706	48	0	0.707	47	0
Viet Nam	0.707	0.819	0.466	0.664	110	115	0.658	110	0.596	109	-1	0.588	110	0
Yemen	0.550	0.447	0.349	0.449	148	152	0.451	148	0.396	149	1	0.402	148	0
Zambia	0.252	0.664	0.377	0.431	151	156	0.420	155	0.398	148	-3	0.375	155	0
Zimbabwe	0.318	0.833	0.527	0.559	130	124	0.546	131	0.515	128	-2	0.487	131	0

Key: C1, country name; C2, LEB index values from United Nations Development Programme (1999); C3, education index values from United Nations Development Programme (1999) after combining Adult Literacy Rate and Gross Enrolment Rate in the ratio of 2:1; C4, GDP per-capita index values from United Nations Development Programme (1999); C5, HDI values with UNDP maximum/minimum limits of LEB = 85/25 years, EDN = 100/0% and PPP GDP = \$40,000/\$100; C6, country ranking based on HDI values; C7, Borda Count-based country rank; C8, AHDI calculated with UNDP limits used in C5;⁹ C9, AHDI-based country ranking; C10, HDI with new arbitrary set of maximum/minimum limits: LEB = 100/25 years, EDN = 100/0% and GDP per-capita = \$100,000/\$100; C11, HDI-based ranking with new set of limits used in C10; C12, difference in HDI-based ranks using UNDP limits and new set of limits (C11-C6); C13, AHDI calculated with new set of limits used in C10;¹⁰ C14, AHDI-based ranking with new set of limits; C15, Difference in AHDI-based ranks using UNDP data and new set of limits (C9-C14).

The method to calculate the AHDI is as follows.

1. Let:

$l = L_b - L_k$, where L_b is the maximum actual LEB index value, say, of country b , and L_k is the minimum actual LEB index value, say, of country k

$e = E_m - E_n$, where E_m is the maximum actual EDN index value, say, of country m , and E_n is the minimum actual EDN index value, say, of country n

An Adjusted HDI

$g = G_p - G_q$, where G_p is the maximum actual GDP index value, say, of country p , and G_q is the minimum actual GDP index value, say, of country q .

2. Take the minimum of (l, e, g) . Let us suppose that $l < e$ and $l < g$ (i.e. l is the minimum or least value among l, e and g).
3. Then, let $e^* = l/e$ and $g^* = l/g$.
4. Adjust L_j, E_j and G_j as follows.
Since l is minimum, let:

$$aL_j = L_j \text{ for all } j$$

$$aE_j = e^*E_j \text{ for all } j$$

$$aG_j = g^*G_j \text{ for all } j$$
5. $aHDI_j = (aL_j + aE_j + aG_j)/3$
6. Choose $\max_j (aHDI_j)$ and $(HDI)_j$.
7. Let $v = (HDI)_j / \max_j (aHDI_j)$
8. Let $AHDI_j = v(aHDI_j)$
9. Rank countries according to AHDI with higher values getting a better rank.

Let us return to the example given in the earlier third section, and calculate the AHDI on the basis of the method just elaborated.

With limits as $LEB_{\max} = 85$ and $LEB_{\min} = 25$, $EDN_{\max} = 100\%$ and $EDN_{\min} = 0\%$, $GDP_{\max} = \$40,000$ and $GDP_{\min} = \$100$, we have (from Table 1) $l = 0.13$ ($= 0.80 - 0.67$), $e = 0.16$ ($= 0.95 - 0.79$) and $g = 0.0373$ ($= 0.5373 - 0.50$). $\min(l, e, g) = g = 0.0373$. $l^* = g/l = 0.2869$, and $e^* = g/e = 0.340$. $aL_j = l^*(L_j)$, $aE_j = e^*E_j$, and $aG_j = G_j$. With $v = (0.7024/0.3451) = 2.03$, we calculate the AHDI and rankings as presented in Table 4.

With new limits for LEB as $LEB_{\max} = 82$ years and $LEB_{\min}$ of 42 years and other limits remaining unchanged, from Table 2 we find $l = 0.185$ ($= 0.585 - 0.770$), $e = 0.11$ ($= 0.95 - 0.79$) and $g = 0.0373$ ($= 0.5373 - 0.50$). $\min(l, e, g) = g = 0.0373$. $l^* = g/l = 0.2016$, and $e^* = g/e = 0.340$. $aL_j = l^*(L_j)$, $aE_j = e^*E_j$, and $aG_j = G_j$. With $v = (0.6741/0.3204) = 2.10$, we calculate the AHDI and rankings as presented in Table 5.

TABLE 4. AHDI and AHDI-based ranking with UNDP limits

	LEB	EDN	GDP	aL_j	aE_j	$G_j = aG_j$	$aHDI_j$	$AHDI_j$	Rank _j
Country A	65.4	90	2500	0.192	0.306	0.5373	0.3451	0.7024	1
Country B	72.8	79	2000	0.2295	0.2686	0.50	0.3327	0.6771	2

TABLE 5. AHDI and AHDI-based ranking with changed LEB limits

	LEB	EDN	GDP	aL_j	aE_j	$G_j = aG_j$	$aHDI_j$	$AHDI_j$	Rank _j
Country A	65.4	90	2500	0.118	0.306	0.5373	0.3204	0.6741	1
Country B	72.8	79	2000	0.155	0.2686	0.50	0.3079	0.6478	2

This simple example is useful to highlight some important features of the AHDI.

- The method proposed to calculate the AHDI first *scales down* each indicator index value proportionately so as to equalize the spread for *all* indicator index values to that of the *minimum* spread in indicator index values. In Tables 4 and 5, the spread between aL_j , aE_j and aG_j is equal to $\min(l, e, g) = g = 0.0373$. One can also think of this as an implicit adjustment in limits to equalize the spread between the maximum and minimum indicator index values.
- This scaling down of indicator index values will mean that $aHDI_j \leq HDI_j$. For instance, suppose country j has reached index values of 1 for all three indicators. Then if, say, $l < e$ and $l < g$, the $aHDI_j$ will not be equal to 1. The closer l, e, g are in value, the smaller will be the difference between HDI_j and $aHDI_j$. However, since the $aHDI_j$ values can be lowered significantly compared with HDI_j values,⁸ we then *scale up* the $aHDI_j$ to $AHDI_j$ by the constant, v . This makes the AHDI values comparable with the HDI values.
- It is also interesting to note that, given equal weights to LEB, EDN and GDP, the difference between maximum and minimum $aHDI$ values will be $(1/3[\min(l, e, g)])$ and the difference between maximum and minimum AHDI values is $\{v(1/3)[\min(l, e, g)]\}$.
- We have shown that, unlike HDI-based rankings, AHDI-based rankings are invariant to change in limits. At the same time, unlike the Borda Count method, the AHDI meets the objectives of HDI.

Comparing the HDI, the Borda Count method and the AHDI

Table 3 compares the HDI, the Borda Count and the AHDI for United Nations Development Programme (1999) country data. As expected, we find that country ranks do not change with a change in the choice of limits.

Conclusion

We have shown that HDI-based country rankings are not robust or consistent with the choice of fixed maximum and minimum indicator values. One way to overcome this problem is to use a Borda Count-based country ranking method. This method, however, does not meet the objectives of the HDI. The proposed AHDI, like the Borda Count method, is invariant to the choice of fixed maximum and minimum indicator values, and at the same time meets the objectives of the HDI.

Notes

1 Instead of 78 years, as set by the UNDP in its 1990 *Human Development Report*.

2 For countries where $LEB \geq 73$, $L_j \geq 1$.

3 X_i refers to the i th component of the index (health, education, etc.).

An Adjusted HDI

- 4 $[(LEB_{max} - LEB_{min})/100]$, $[(EDN_{max} - EDN_{min})/100]$ and $\log(GDP_i) = \{0.01[\log(GDP_{max}) - \log(GDP_{min})] + \log(GDP_{min})\}$. Since income is indexed on a logarithmic scale, the amount of income that contributes 0.01 to G_i will increase with the absolute level of income.
- 5 For ranking using the Borda Count, each country is first ranked by indicators separately. A score for each country is computed as the sum of indicator ranks. Overall country ranks are assigned on the basis of these scores, where a lower score means a better country rank. A change in indicator limits will change indicator index values for the countries but not their ranking, which makes the Borda Count-based ranking independent of the choice of limits.
- 6 Such as 0.6 years $LEB \pm 1\%$, $EDN = \$150$, and $\$123$, GDP.
- 7 The difference in values and ranks reported in United Nations Development Programme (1999) and Table 3 are due to differences in decimal places considered for rounding off.
- 8 The large change in HDI values from Table 1 is because of the small value of g . This may not be the case for a larger sample where differences in the values of l , e and g are not so large. However, the scaling up of aHDI to AHDI by v takes care of this problem.
- 9 With UNDP limits, we can see that in Table 3 (from C2, C3 and C4), $l = 0.713,334$ ($= 0.916,66 - 0.203,333$), $e = 0.848$ ($= 0.9933 - 0.1453$), $g = 0.721,218,681$ ($= 0.956,718,192 - 0.235,499,511$). Therefore, $\min(l, e, g) = l = 0.713,334$. $e^* = l/e = 0.841,195,754$ and $g^* = 0.989,067,558$. For, say, Canada, $aL_j = L_j = 0.90$, $aE_j = e^*(0.99) = 0.832,783,7$ and $aG_j = g^*(0.90) = 0.890,160,7$. $aHDI_j$ for Canada $= 0.874,314,8$. $v = 0.932/0.874,314,8 = 1.065,977,6$.
- 10 With the change in limits, we must first determine the new L_j , E_j and G_j values (this is not shown in Table 3). We will find the following: $l = 0.570,666$ ($= 0.733,333 - 0.162,667$), $e = 0.848$ ($= 0.9933 - 0.1453$), $g = 0.625,552$ ($= 0.829,813 - 0.204,261$). Therefore, $\min(l, e, g) = l = 0.570,666$. $e^* = l/e = 0.672,955,188$ and $g^* = l/g = 0.912,259,892$. Then, for, say, Canada, $aL_j = L_j = 0.72$, $aE_j = 0.666,226$ and $aG_j = 0.715,15$. $aHDI_j$ for Canada is $0.700,458$. $v = 0.831,311/0.700,458 = 1.186,810,6$.

References

- Atkinson, G., Dubourg, R., Hamilton, K., Munasinghe, M., Pearce D. and Young, C. (1999) *Measuring Sustainable Development: Macroeconomics and the Environment*, Edward Elgar, Cheltenham.
- Dasgupta, P. (1993) *An Enquiry into Wellbeing and Destitution*, Oxford University Press, Oxford.
- Kelley, A.C. (1991) 'The Human Development Index: "handle with care"', *Population and Development Review*, 17(2), pp. 315-324.
- Ray, Debraj (1998) *Development Economics*, Oxford University Press, New Delhi.
- Sen, Amartya K. (1999) Assessing human development — special contribution, *UNDP Human Development Report 1999*, Oxford University Press, New Delhi.
- United Nations Development Program (1999) *Human Development Report 1999*, Oxford University Press, New York.